# Observational Evidence for Extra Dimensions from Dark Matter

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Recent astronomical observations of systems of dark matter, which have been cited as providing possible support for self-interacting cold dark matter, may provide evidence for the extra dimensions predicted by superstring scenarios. We find that the properties of the required dark matter self-interaction are precisely the consequences of a world with 3 large extra dimensions of size ∼1nm, where gravity follows the $r^{-5}$ law at scales below ∼1nm. From the cross sections measured for various dark matter systems, we also constrain the mass of dark matter particles to be $m_x \sim 3 \times 10^{-16}$ proton mass, consistent with the mass of axions.

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String theory is a theory of all fundamental physical interactions. While in principle able to explain all phenomena, it has in practice been very challenging to find any observable predictions. One generic prediction is the existence of extra dimensions in addition to our familiar 3-dimensional space. These extra dimensions had been thought to be extremely tiny (of order the Planck length $\sim 10^{-33}$cm), until in recent years the idea of large extra dimensions was proposed to address the "hierarchy problem"—a problem associated with the factor of the $10^{17}$ huge difference between the Planck scale and the weak interaction scale. In the Arkani-Hamed, Dimopoulos and Dvali (ADD) scenario, the electro-weak and Planck energies are the same, and the large extra dimensions explain the apparent discrepancy in strength when measured on macroscopic scales [1]. In the context of string cosmology, the fundamental scale would be $\sim$ 1TeV, corresponding to $10^{-17}$cm. There are $3+1$ cosmological size dimensions, $n$ large extra dimensions, and $6-n$ fundamental scale dimensions.

The size $R$ of these large extra dimensions depends on the number of large extra dimensions $n$ . Gravity would start to deviate from Newton's inverse square law at small distance scales $r < R$. For $n = 2$, $R \sim$ 1mm. This opens a new window for experimental tests of string theory and searches for extra dimensions, by precise measurements of the gravitational force at sub-mm scales. Tremendous efforts have been made in the past few years in testing Newton's inverse square law at small scales and searching for evidence of the large extra dimensions. Currently, the measurements are reaching micron scales and no deviation from Newton's law has been found from ∼1cm down to $\sim 10^{-3}$cm [2, 3, 4, 5].

While string scenarios and extra dimensions have not yet been tested experimentally so far in the laboratory, recent astronomical observations of dark matter, on the other hand, may shed light on the issue. It should be noted that any connection of string scenarios to observable phenomena would be an exciting possibility deserving further investigation.

The existence of dark matter was first realized by Zwicky nearly 70 years ago [6], and it is now well established that dark matter constitutes the major matter component of the universe [7]. Although its nature remains a mystery, most cosmologists and particle physicists believe that dark matter is likely to be a new species of elementary particle that is neutral, long-lived, cold (or nonrelativistic), and collisionless (i.e. dark matter particles have very little interactions with themselves as well as with ordinary matter). This "standard" picture of collisionless cold dark matter (CCDM) has gained great success in explaining the origin and evolution of cosmic structures on large scales [7, 8, 9, 10].

However, the CCDM model is facing a potential challenge in recent years from observations on galactic and sub-galactic scales [8, 11]. Numerical simulations of the CCDM model predict that the density profiles of dark matter halos should exhibit a cuspy core in which the density rises sharply as the distance from the center decreases [12]. In contrast, observations of systems of dark matter, ranging from dwarf galaxies [13, 14, 15, 16], low surface brightness galaxies [17, 18] to galaxies comparable in mass with the Milky Way [19], indicate that the central density profiles are probably much less cuspy than predicted. Clusters of galaxies also seem to reveal a near isothermal core [20, 21], although there is considerable scatter [22].

The plausible discrepancies between theory and observations, although still vigorously debated, have stimulated many attempts to understand the nature of dark matter and to modify the CCDM model, among which one of the more popular schemes is the self-interacting cold dark matter model. As proposed by Spergel and Steinhardt, the above conflicts can be readily resolved if the cold dark matter particles are self-interacting with a large scattering cross section $\sigma_{xx}/m_x \approx 8 \times 10^{-(25-22)}$cm$^2$/GeV, where $m_x$ is the mass of dark matter particles [11].

Follow-up work by many authors, however, suggests that the issue may be more complicated. In galaxy

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clusters, dark matter was found to probably have a much weaker self-interaction. Gravitational lensing studies of clusters have placed an upper limit $\sigma_{xx}/m_x < 10^{-25.5}\text{cm}^2/\text{GeV}$ [23]. Detailed X-ray observations of clusters revealed that $\sigma_{xx}/m_x < 2 \times 10^{-25}\text{cm}^2/\text{GeV}$ [24, 25], and a joint X-ray/weak-lensing study showed that $\sigma_{xx}/m_x < 2 \times 10^{-24}\text{cm}^2/\text{GeV}$ [26]. From numerical simulations, Yoshida *et al.* found that the cross sections needed to produce good agreement with galaxies turned out to produce galaxy cluster cores that were too large and too round to be consistent with observation [27].

These lines of evidence might suggest that, instead of been a fixed value, the dark matter self-interaction cross section as proposed by Spergel and Steinhardt, is likely to vary in different dark matter systems—smaller systems (like dwarf galaxies) tend to have larger cross sections whereas more massive systems (like galaxy clusters) tend to have smaller cross sections. Indeed, Firmani *et al.* have constrained the cross section from various dark matter systems ranging from dwarf galaxies to galaxy clusters, and proposed a relation [28]

$$\frac{\sigma_{xx}}{m_x} \approx 4 \times 10^{-25} \left( \frac{100 \text{ km s}^{-1}}{v} \right) \text{ cm}^2/\text{GeV} \qquad (1)$$

where $v$ is the velocity dispersion of the dark matter system. Note that more massive systems (like galaxy clusters) have higher velocity dispersions while less massive systems (like dwarf galaxies) have lower velocity dispersions.

The nature of this self-interaction between dark matter particles is unknown. Its strength generally must be put in by hand. We note that the scattering cross section in Eq. (1) decreases with increasing velocity, which is characteristic of long-range forces (like gravity or Coulomb forces). On the other hand, as addressed by ADD, gravity has only been accurately measured in the $\sim$1cm range but has been extrapolated to small distance scales on the assumption that gravity is unmodified over the 33 orders of magnitude from $\sim$1cm down to the Planck length $\sim 10^{-33}$cm. Will gravity still follow the conventional Newtonian $r^{-2}$ law at extremely small distance scales? Does the dark matter self-interaction have anything to do with the (microscopic) asymptotic behavior of gravity?

Here we propose that if gravity deviates from Newton's inverse square law at sub-mm scales and varies as $r^{-(2+n)}$, as suggested by ADD, then the strength of gravity would be greatly enhanced at small distance scales and hence could *naturally* provide the self-interaction for dark matter particles, without introducing a new interaction which would otherwise seriously complicate the Standard Model. The self-interaction between dark matter particles, if true, may have strong implications for the modification of gravity at small distance scales that experimental physicists have been searching for during the past few years. Also, from the cross sections determined for various dark matter systems, we can constrain how gravity varies with distance.

We assume that at small distance scales below the "critical radius" $R$, gravity starts to deviate from the Newtonian $r^{-2}$ law and takes the general form:

$$F = \alpha \frac{GMm}{r^{2+n}}, \qquad (2)$$

where $\alpha$ is a constant with dimension $[\text{length}]^n$, $G$ is the gravitational constant, $M$ and $m$ are the masses of the two particles. The value of $\alpha$ is easily determined to be $\alpha = R^n$, from the boundary condition that at $r = R$, $\alpha \frac{GMm}{r^{2+n}} = \frac{GMm}{r^2}$,

When dark matter particles with a mean relative velocity $u$ approach close enough to each other, gravitational scattering may cause large deflection angles. For particles moving in the central force field described by Eq. (2), the elastic scattering cross section is given by [29]

$$\sigma = \pi A \left( \frac{\alpha G m_x}{u^2} \right)^{\frac{2}{n+1}}, \qquad (3)$$

where $A = [(n+1)/(n-1)]^{(n-1)/(n+1)}$ for $n > 1$, and $A = 1$ for $n = 1$. The value of $A$ is close to 1.4 for $2 \leq n \leq 5$. Taking into account the relation $u^2 = 2v^2$ between the mean relative velocity and the velocity dispersion, we can then estimate the elastic scattering cross section between dark matter particles as:

$$\sigma_{xx} = \pi A \left( \frac{\alpha G m_x}{2v^2} \right)^{\frac{2}{n+1}}, \qquad (4)$$

where $A = 1$ for $n = 1$, and $A \approx 1.4$ for $2 \leq n \leq 5$. Combining Eqs (1) and (4) we find that there is only one solution which is the $n = 3$ case, corresponding to 3 large extra dimensions. The case of $n \neq 3$ was excluded, because Eqs (1) and (4) result in an unrealistic solution where the value of $m_x$ varies with $v$.

The above calculation of the elastic scattering cross section was based on classical mechanics. However, for the problem discussed in this letter, the de Broglie wavelength of the particles is much greater than the length scale at which the particles interact with each other. This can be seen from the smallness of the dark matter particle mass (shown in following paragraphs). Hence, a quantum mechanical treatment is required. Fortunately, for the elastic scattering cross section in the central force field as in our case, quantum mechanics gives similar results as classical mechanics.

From a detailed calculation, Vogt and Wannier have shown that the quantum mechanical scattering cross section is exactly twice the classical value, for a central force field of the $r^{-5}$ form [30], corresponding to our $n = 3$ case. For the general case of $n$, Joachain has shown that the boson-boson identical particle scattering cross section in s-wave is twice the classical cross section, while the s-wave scattering cross section of identical fermions is four times smaller than the boson-boson scattering [31]. Note that our case is in the low energy regime where the s-wave scattering dominates. Therefore, our classical

constraints on $n$ from Eqs (1) and (4) are still valid in the quantum mechanical regime. A factor of a few difference in $\sigma_{xx}$ does not change the value of $n$ we have derived above.

Correcting Eq. (4) by a factor of two, we have the quantum mechanical cross section in the $n = 3$ case:

$$\sigma_{xx} = 2\pi \frac{(\alpha G m_x)^{\frac{1}{2}}}{v}, \quad \text{for } n = 3. \tag{5}$$

Combining Eqs (1) and (5) we obtain $m_x = 3\times10^5\alpha$ GeV, with $\alpha = R^n$. According to ADD, the size $R$ of the extra dimensions can be expressed as $R \sim 10^{\frac{30}{n}-17}$cm. We therefore find that $R \sim 10^{-7}$cm and the dark matter particle mass $m_x \sim 3 \times 10^{-16}$GeV.

By attributing the dark matter self-interaction to modified gravity at small distance scales as suggested in the ADD scenario, we have obtained the following results:

(1) We have avoided the introduction of a new fine-tuned interaction in the Standard Model by using an existing physical scenario.

(2) The number of large extra dimensions has been constrained and found to be $n = 3$.

(3) The size of the large extra dimensions was found to be of order $R \sim 10^{-7}$cm, below which gravity deviates from Newton's inverse square law and varies as $r^{-5}$.

(4) The mass of dark matter particles was constrained to be $m_x \sim 3 \times 10^{-16}$ proton mass—falling into roughly the mass range of the axion which has been proposed as a dark matter candidate (see e.g. [32]).

This ties together the large extra dimension string scenario proposed solely for particle physics and the dark matter halo structure puzzle in astrophysics, without introducing any new or fine-tuned parameters. The standard self-interacting dark matter solution to the problem requires fine-tuned cross sections, and does not explain the scale-dependent behavior. Interaction cross sections are enhanced over the characteristic electro-weak values because of the relatively slow motions of particles in dark matter halos.

It should be noted that the issue of self-interacting dark matter is still under debate. In more massive systems like galaxy clusters, the self-interaction cross section appears to be small, while in less massive systems like low surface brightness galaxies and dwarf galaxies, the larger cross section could be due to mis-interpretation of the observed data. Further studies are needed to confirm the self-interacting dark matter model. The $n$ and $m_x$ values we obtained from the empirical formula Eq. (1) is sensitive to the form of cross section determined from observations. A cross section different from Eq. (1) will result in different $n$ and $m_x$: the dependance of $\sigma_{xx}$ on the velocity dispersion $v$ determines the value of $n$, and $m_x$ is also affected by the coefficients in Eq. (1). Furthermore, $m_x$ is sensitive to the exact form of $R$ as a function of $n$ given by ADD. Overall, our constraints on the extra dimensions relies on an accurate determination of the self-interaction cross section.

Nevertheless, we are suggesting a potentially new connection between string scenarios and astrophysical phenomena. The ADD scenario with 3 large extra dimensions naturally explains the velocity-dependent cross section (as in Eq. (1)) for self-interacting dark matter, and predicts the number and size of extra dimensions, and also the dark matter particle mass. The current limits on gravity at small scales are problematic for the original $n = 2$ proposal in the ADD scenario, but our model predicts deviations from Newtonian gravity at the nanometer scale. The generic prediction of the ADD scenario is a wealth of quantum gravity phenomena at the LHC.

Ordinarily, string scenarios are evaluated on the basis of aesthetics and heuristic mathematical arguments. For example, it has been speculated that the large extra dimensions might have instabilities. The empirically appealing aspect of the ADD scenario is its connection to observable phenomena, which allows a scientific evaluation, via test or falsification.

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